9. EXERCISE SHEET, RETURN DATE JULY 9./10.

Exercise 1

Instead of

$$K_d u_d = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

we want to solve the preconditioned system

$$B_d^T K_d B_d \bar{u}_d = B_d^T \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}, \quad u_d = B_d \bar{u}_d$$

with

$$B_1 = (1)\,,\; B_2 = \begin{pmatrix} 1 & 1 & \\ & 2 & \\ & 1 & 1 \end{pmatrix} B_3 = \begin{pmatrix} 1 & 1 & & 1 & & & \\ & 2 & & 2 & & & \\ & 1 & 1 & 3 & & & \\ & & & 4 & & & \\ & & & 3 & 1 & 1 & \\ & & & 2 & & 2 & \\ & & & 1 & & 1 & 1 \end{pmatrix},$$

$$B_4 = \begin{pmatrix} 1 & 1 & & 1 & & & & 1 & & & & & & & \\ & 2 & & 2 & & & & 2 & & & & & & & \\ & 1 & 1 & 3 & & & & 3 & & & & & & & \\ & & & 4 & & & & 4 & & & & & & & \\ & & & 3 & 1 & 1 & & 5 & & & & & & & \\ & & & 2 & & 2 & & 6 & & & & & & & \\ & & & 1 & & 1 & 1 & 7 & & & & & & & \\ & & & & & & & 8 & & & & & & & \\ & & & & & & & 7 & 1 & 1 & & 1 & & & \\ & & & & & & & 6 & & 2 & & 2 & & & \\ & & & & & & & 5 & & 1 & 1 & 3 & & & \\ & & & & & & & 4 & & & & 4 & & & \\ & & & & & & & 3 & & & & 3 & 1 & 1 & \\ & & & & & & & 2 & & & & 2 & & 2 & \\ & & & & & & & 1 & & & & 1 & & 1 & 1 \end{pmatrix}, \ldots$$

(The idea for these B_d comes from the 1986 paper 'On the Multi-Level Splitting of Finite Element Spaces' by Harry Yserentant.)

Implement this with your gradient and cg algorithm and plot again the individual iterations. Compare with the previous homework sheet.

Exercise 2

Prove that $B_d^T K_d B_d = \begin{pmatrix} B_{d-1}^T K_{d-1} B_{d-1} & 0 & 0 \\ 0 & 2^d & 0 \\ 0 & 0 & B_{d-1}^T K_{d-1} B_{d-1} \end{pmatrix}$ and calculate its condition number.

Exercise 3

By using the definitions introduced in the lecture, determine the type of the following differential equations (elliptic, parabolic, hyperbolic). Allways we have $u : \mathbb{R}^{\cdots} \to \mathbb{R}$

$$\dot{u} = u'' \,(\text{time dependent diffusion in one dimension})$$

$$\triangle u - \nabla u = f$$

$$\dot{u} = \triangle u \,(\text{time dependent diffusion})$$

$$\ddot{u} = \triangle u \,(\text{wave equation})$$

$$\triangle u = 0 \text{ (Laplace equation)}$$

Exercise 4 (optional)

Implement an implicit scheme for the problem in exercise 3 of the 4th homework sheet.

Exercise 5

Continue preparing for your exams.